

# Aharonov-Bohm oscillations of a particle coupled to dissipative environments

F. Guinea

*Instituto de Ciencia de Materiales de Madrid, CSIC, Cantoblanco, E-28049 Madrid, Spain.*

(Dated: February 6, 2008)

The amplitude of the Bohm-Aharonov oscillations of a particle moving around a ring threaded by a magnetic flux and coupled to different dissipative environments is studied. The decay of the oscillations when increasing the radius of the ring is shown to depend on the spatial features of the coupling. When the environment is modelled by the Caldeira-Leggett bath of oscillators, interference effects are suppressed beyond a finite length, even at zero temperature. A non trivial renormalization of the Aharonov-Bohm oscillations is also found when the particle is coupled by the Coulomb potential to a dirty electron gas. A finite renormalization of the Aharonov-Bohm oscillations is obtained for other models of the environment.

## I. INTRODUCTION

Phase coherence in metallic systems has been extensively studied since experiments suggested that the dephasing time,  $\tau_\phi$ , seemed to saturate to a constant value at low temperatures<sup>1</sup>, in apparent contradiction with the accepted theory<sup>2,3</sup>. It has been argued that voltage fluctuations lead to a dephasing time consistent with saturation at low temperatures<sup>4,5</sup>, although related calculations lead to different results<sup>6,7</sup> (see also<sup>8</sup>).

Comparison between the different calculations of the dephasing time of low energy electrons in metals is obscured by the various approximations required to deal with the interactions and quenched disorder. The cause of dephasing, however, is the existence of a dynamic environment interacting with the electrons. A simpler situation is presented when the environment is different from the particles whose dephasing is being studied. Even if that is the case, in a many particle system the environment induces interactions between the particles. Thus, the simplest case when dephasing at low temperatures can be studied is that of a single particle coupled to an external dissipative environment. The problem can also be relevant to studies of quantum effects in heavy particles at metallic surfaces.

We will study here the amplitude of the Aharonov-Bohm oscillations of the particle moving around a ring of radius  $R$  threaded by a magnetic flux  $\Phi$ . This quantity provides information about the suppression of quantum interference due to the environment. We will not attempt to define a dephasing time. On the other hand, the dependence of the Aharonov-Bohm oscillations on the radius of the ring allows us to define, in certain cases, a length scale,  $R_\phi$ , beyond which the oscillations decay exponentially or have a gaussian dependence on the radius.

The simplest quantity which can be studied which depends on the flux is the free energy. At zero temperature, and in the absence of dissipative effects, the amplitude of the oscillations of the energy as function of  $\Phi$  is of order  $\hbar^2/(MR^2)$ , where  $M$  is the mass of the particle. The power law dependence of this scale on the length of the path of the particle can be interpreted as the absence of a typical length for the suppression of quantum coherence effects, at zero temperature.

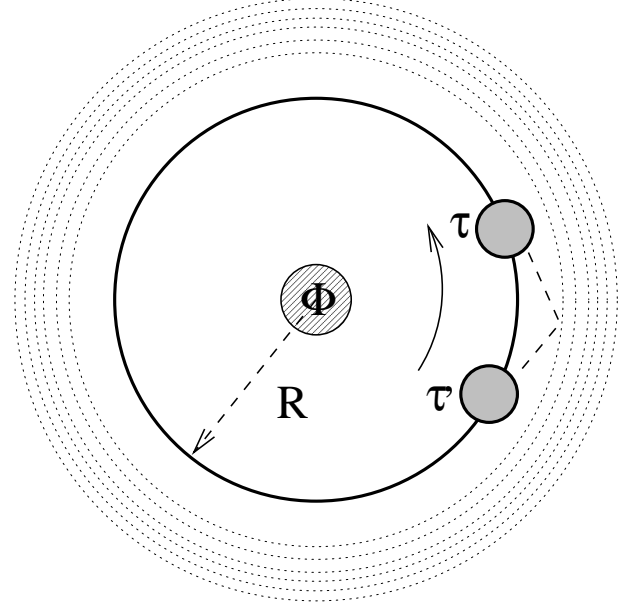


FIG. 1: Schematic picture of the system studied in the text. A particle interacting with a dissipative environment moves around a circle of radius  $R$ . The circle is threaded by a magnetic flux,  $\Phi$ .

In the present work, we estimate how this amplitude is changed when the particle is coupled to a dissipative bath. The next section presents the model used to analyze this problem. Section III discusses specific results for five types of dissipative environments: i) The Caldeira-Leggett harmonic bath<sup>9</sup> (a closely related aspect of this model was analyzed in<sup>10</sup>), ii) Dissipation with a periodic spatial dependence (the dissipative quantum rotor<sup>11</sup>), iii) Dissipation with a gaussian spatial dependence (the DVD model<sup>12</sup>), iv) Dissipation induced by a local coupling to the low energy modes of a clean metallic system<sup>13</sup>, and v) Dissipation induced by the excitations of a dirty metallic system. Section IV presents the conclusions.

## II. THE MODEL.

We assume that the degrees of freedom in the environment can be integrated out, leading to retardation effects in the equations of motion of the particle that we are interested in. These effects are best studied using the path integral formulation of quantum mechanics<sup>14</sup>. Then, the action associated to each path of the particle, in the absence of a magnetic flux, can be written as:

$$\frac{S}{\hbar} = \int d\tau \frac{M}{2\hbar} \left( \frac{\partial X}{\partial \tau} \right)^2 + \int d\tau d\tau' K[X(\tau) - X(\tau'), \tau - \tau'] \quad (1)$$

where  $M$  is the mass of the particle, and  $X(\tau)$  is its position at time  $\tau$ . The kernel  $K(X, \tau)$  includes the information about the environment and its interaction with the particle. At long times, one has:

$$K(X, \tau) \approx \frac{\mathcal{K}(X)}{|\tau|^2} \quad (2)$$

where  $\mathcal{K}$  depends only on the spatial coordinates.

A simple choice is given by the Caldeira-Leggett model<sup>9,14</sup>:

$$K(X, \tau) = \frac{\gamma}{2\pi\hbar} \frac{X^2}{\tau^2} \quad (3)$$

and  $\mathcal{K}(X) = (\gamma X^2)/(2\pi\hbar)$ , where  $\gamma$  is the friction coefficient which describes the dynamics of the particle when  $\hbar \rightarrow 0$ .

Two extensions of the kernel in eq.{3} are<sup>11,12</sup>:

$$\begin{aligned} K(X, \tau) &= \frac{\alpha}{2\pi} \frac{\sin^2[X/(4\pi L)]}{\tau^2} \\ K(X, \tau) &= \frac{\gamma l^2}{2\pi\hbar} \frac{e^{-X^2/(2l)}}{\tau^2} \end{aligned} \quad (4)$$

In the first case, the macroscopic friction coefficient is  $\gamma = (\hbar\alpha)/(4\pi L)^2$ . In the second case,  $l$  is a length which defines the spatial range of the interactions mediated by the environment.

When the coupling between the particle and the environment is weak, one can use perturbation theory. In this work, we will study the coupling of the particle to a metallic system by means of a local potential of strength  $U$  and range  $a$ :

$$\mathcal{H}_{int} = U \int d^3\mathbf{r} \mathcal{F} \left( \frac{|\mathbf{r} - \vec{\mathbf{R}}|}{a} \right) \rho(\mathbf{r}) \quad (5)$$

where  $\vec{\mathbf{R}}$  is the coordinate of the particle,  $\rho(\mathbf{r})$  is the density operator of the electron gas, and  $\mathcal{F}(u) \sim 0$  if  $u \gg 1$ . This coupling leads to a retarded interaction along the path taken by particle. Using second order perturbation theory, this interaction can be cast as<sup>13</sup>:

$$S_{int} = \frac{U^2}{2\pi\hbar} \int d\tau d\tau' d^3\mathbf{r} d^3\mathbf{r}' d\omega d^3\mathbf{k} \mathcal{F} \left( \frac{|\vec{\mathbf{R}}(\tau) - \mathbf{r}|}{a} \right) \mathcal{F} \left( \frac{|\vec{\mathbf{R}}(\tau') - \mathbf{r}'|}{a} \right) e^{i\mathbf{k}(\mathbf{r} - \mathbf{r}')} e^{i\omega(\tau - \tau')} \chi(\vec{\mathbf{k}}, \omega) \quad (6)$$

where  $\chi(\vec{\mathbf{k}}, \omega)$  is the density-density correlation function of the metal.

The coordinate  $X$  of the particle will be restricted to lie within a circle of radius  $R$ , so that the motion can be also described in terms of the angle  $\theta(\tau)$ , as schematically shown in Fig.[1]. In terms of this angle, we have:

$$|\vec{\mathbf{R}}(\tau) - \vec{\mathbf{R}}(\tau')| = 2R \sin \left[ \frac{\theta(\tau) - \theta(\tau')}{2} \right] \quad (7)$$

The action, eq.{1}, can be expanded in circular harmonics as:

$$\frac{S}{\hbar} = \int d\tau \frac{MR^2}{2\hbar} \left( \frac{\partial \theta}{\partial \tau} \right)^2 + \int d\tau d\tau' \sum_n \alpha_n \frac{\sin^2 \left\{ \frac{n[\theta(\tau) - \theta(\tau')]}{2} \right\}}{|\tau - \tau'|^2} \quad (8)$$

where the  $\alpha_n$ 's are dimensionless constants, given by:

$$\alpha_n = \frac{1}{2\pi} \int d\theta e^{in\theta} \mathcal{K} [2R \sin(\theta/2)] \quad (9)$$

The action in eq.{8} resembles closely the quantum ro-

tor model<sup>15</sup> studied extensively in relation to Coulomb blockade in normal tunnel junctions<sup>11</sup>. In the present case, the short range interaction is related to the energy scale  $\hbar^2/(2MR^2)$ . When studying the quantum rotor model in the context of mesoscopic junctions, this scale corresponds, at the same time, to the charging energy of the junction,  $E_C$ , and to the upper cutoff in the spectrum of the environment coupled to the variable under study. In the present case, the equivalent to  $E_C$  depends on the radius of the ring,  $R$ , and the two scales should be kept separate. Thus, and following the conventional notation for tunnel junctions, the model in eq.{8} contains two scales,  $E_C = \hbar^2/(2MR^2)$ , and an energy,  $\Lambda_0$ , which defines the short time cutoff in the kernel in eq.{2}. We will assume that  $E_C \ll \Lambda_0$ .

In order to analyze the problem, we extend the Renormalization Group approach initially discussed in<sup>15</sup> to the action in eq.{8}. We need to consider the scaling of the parameters  $\{\alpha_n\}$ , and, for completeness, we also consider the renormalization of the dimensionless coupling  $\tilde{E}_C = E_C/\Lambda$  due to the high energy excitations of the environment. We lower the effective high energy cutoff from  $\Lambda$  to  $\Lambda - d\Lambda$ , and rescale the dimensionless parameters  $\tilde{E}_C$  and  $\{\alpha_n\}$ . This leads to the equations:

$$\begin{aligned} \frac{\partial \tilde{E}_C}{\partial l} &= \tilde{E}_C - c_1 \sum_n n^2 \alpha_n \tilde{E}_C^2 \\ \frac{\partial \alpha_n}{\partial l} &= -\frac{1}{2\pi^2} \frac{n^2 \alpha_n}{c_2 \tilde{E}_C^{-1} + \sum_m m^2 \alpha_m} \end{aligned} \quad (10)$$

where  $l = -\log(\Lambda)$  and  $c_1$  and  $c_2$  are constants of order unity. The scaling equation for  $\tilde{E}_C$  ceases to be valid when  $\tilde{E}_C \sim 1$ . The renormalization of the  $\{\alpha_n\}$ 's is only significant when  $\tilde{E}_C \sim 1$ . In addition, the equations for the  $\{\alpha_n\}$ 's must be halted when  $\sum_n n^2 \alpha_n \sim 1$ . We can write the second equation in {10} as:

$$\frac{\partial \sum_n n^2 \alpha_n}{\partial l} = -\frac{1}{2\pi^2} \frac{\sum n^4 \alpha_n}{c_2 \tilde{E}_C^{-1} + \sum n^2 \alpha_n} \quad (11)$$

The equation which determines the flow of  $\tilde{E}_C$  shows two regimes, depending on whether

$$\kappa = \frac{(\sum_n n^2 \alpha_n^0)^2}{\sum n^4 \alpha_n^0} \tilde{E}_C^0 \quad (12)$$

is smaller or greater than one. When  $\kappa \ll 1$ , the absolute value of  $E_C$  is not changed by the renormalization of the modes with energies between  $\Lambda_0$  and  $E_C$  itself. In this case,  $E_C$  defines the cutoff of an effective theory where the only couplings left are the  $\{\alpha_n\}$ 's, in a similar fashion to the usual case studied in<sup>15</sup>. This is the situation which is most relevant to the calculations to be performed in the following. The effective low energy scale is the given by:

$$E_C^{ren} \sim E_C \exp \left[ -2\pi^2 \frac{(\sum n^2 \alpha_n)^2}{\sum n^4 \alpha_n} \right] \quad (13)$$

If  $\kappa \gg 1$ , the renormalization of  $\tilde{E}_C$  is determined by the coupling to the environment, and only when this coupling flows towards zero  $\tilde{E}_C$  can approach unity. the scale at which  $\tilde{E}_C \sim 1$  implies that:

$$E_C^{ren} \sim \Lambda_0 e^{-\tilde{E}_C^0 \kappa} e^{-\kappa} \sim \Lambda_0 e^{-\lambda/\tilde{E}_C^0} \quad (14)$$

where  $\lambda$  is a constant.

The contribution to the action from the magnetic flux threading the circle is a topological term, which can be written as:

$$\frac{\delta S}{\hbar} = i \frac{\Phi}{\Phi_0} \int d\tau \frac{\partial \theta}{\partial \tau} \quad (15)$$

where  $\Phi_0$  is the quantum unit of flux. This term enters in the effective action in the same way a gate voltage is included in the corresponding problem of tunneling in a small junction, which has also been studied in the literature<sup>16,17</sup>. It is known that the fluctuations in the free energy at low temperatures are determined by the renormalized value of  $E_C^{ren}$ . In the following, we will assume that the amplitude of the Aharonov-Bohm oscillations in the free energy of a particle moving around a ring are also determined by  $E_C^{ren}$ , as calculated using the scaling equations {10}. This scale is given in eq.{31} or eq.{14}, where  $E_C^0 \approx \frac{\hbar^2}{MR^2}$ .

### III. RESULTS.

#### A. Caldeira-Leggett model.

The dynamics of a quantum particle around a ring, using the Caldeira-Leggett bath of oscillators as a model for the environment, has been considered in<sup>10</sup>. The analysis presented here of the Aharonov-Bohm oscillations is consistent with the results in<sup>10</sup>.

For the present model, the coefficients in the harmonic expansion in eq.{8} reduce to:

$$\alpha_n = \delta_{n,1} \frac{\gamma R^2}{\hbar} = \alpha \quad (16)$$

and the parameter in eq.{12} is  $\kappa \sim (\hbar\gamma)/(M\Lambda_0) \ll 1$ . We recover the quantum rotor in its simplest version. Using eq.{13}, we find:

$$E_C^{ren} \sim E_C e^{-2\pi^2 \alpha} \sim \frac{\hbar^2}{MR^2} e^{-(2\pi^2 \gamma R^2)/\hbar} \quad (17)$$

The Aharonov-Bohm oscillations will show a gaussian decay as the radius of the ring is increased. Thus, quantum interference effects are suppressed beyond certain length,  $R_\phi \sim \sqrt{(\hbar/\gamma)}$ . This suppression of the Aharonov-Bohm oscillations are in qualitative agreement with the vanishing of the Landau diamagnetism of a particle interacting with a Caldeira-Leggett bath of oscillators in a magnetic field<sup>18</sup>, at zero temperature. In the language used in<sup>18</sup>,

our results suggest that, in a constant magnetic field  $B$ , the properties of the system at zero temperature are a function of the dimensionless ratio  $(\gamma r_c^2)/\hbar = (\gamma c)/(eB)$ , where  $r_c$  is the cyclotron radius. This gaussian suppression of the Aharonov-Bohm oscillations at zero temperature is consistent with the similar suppression of interference effects between time reversed paths discussed in<sup>10</sup>.

Note that, as  $\alpha \sim O(R^2)$ , the scaling equations, {10} are valid for large values of  $R$ .

### B. The dissipative quantum rotor.

We now consider the retarded interaction described in eq.{4}. The spatial dependence of the kernel allows for only one Fourier component, so that the decomposition needed in eq.{8} becomes:

$$\alpha_n = \alpha \delta_{n,n_0} \quad (18)$$

where  $\alpha$  is the dimensionless constant in eq.{4}, and  $n_0 = R/L$ , where  $L$  is the period of the kernel. The parameter in eq.{12} is  $\kappa \sim (\alpha \hbar^2)/(ML^2 \Lambda_0) \ll 1$ .

Using eq.{13}, we find:

$$E_C^{ren} \sim \frac{\hbar^2}{MR^2} e^{-2\pi^2 \alpha} \quad (19)$$

Hence, the Aharonov-Bohm oscillations show the dependence on  $R$  as in the non dissipative case, although the coupling to the environment leads to a finite reduction of the amplitude.

### C. Coupling via a kernel with a gaussian dependence on distance.

This model<sup>12</sup> is defined in terms of a kernel, eq.{2},

$$\mathcal{K}(X) = \frac{\gamma l^2}{2\pi} e^{-X^2/(2l^2)} \quad (20)$$

where  $\gamma$  is the friction coefficient, and  $l$  is a length which characterizes the spatial correlations in the bath.

We now use  $X = 2R \sin(\theta/2)$ , and perform the Fourier transform in eq.{9} using the saddle point approximation, to obtain:

$$\alpha_n \approx \frac{\gamma l^2}{\hbar} \left( \frac{l}{R} \right) e^{-(n^2 l^2)/(2R^2)} \quad (21)$$

The parameter  $\kappa$  in eq.{12} is:

$$\kappa \sim \frac{\gamma l^2}{\hbar} \frac{\hbar^2 \Lambda_0}{MR^2} \quad (22)$$

where  $\Lambda_0$  is the high energy cutoff of the environment. We have  $\kappa \ll 1$  when  $R \gg l$ . The renormalization of the Aharonov-Bohm oscillations is given by:

$$E_C^{ren} \sim \frac{\hbar^2}{MR^2} e^{-(2\pi^2 \gamma l^2)/\hbar} \quad (23)$$

The amplitude of the Aharonov-Bohm oscillations is reduced by a finite factor.

### D. Coupling to a clean electron gas.

The density-density correlation function of a three dimensional electron gas can approximately be written, at low frequencies, as:

$$\chi(\vec{k}, \omega) \approx \frac{|\hbar\omega|}{2\pi^2(\hbar^2/m)^2|\vec{k}|} \theta \left( 1 - \frac{|\vec{k}|}{2k_F} \right) \quad (24)$$

where  $m$  is the electron mass,  $k_F$  is the Fermi wavevector, and  $\theta(u)$  is the step function. The high energy cutoff is  $\Lambda_0 \sim E_F \sim (\hbar^2 k_F^2)/(2m)$ , where  $E_F$  is the Fermi energy. The interaction term in the effective action, eq.{6}, can be written as (see eq. {2}):

$$S_{int} \approx \frac{1}{2\pi\hbar} \int d^3\vec{R}(\tau) d^3\vec{R}(\tau') \frac{\mathcal{K}(|\vec{R}(\tau) - \vec{R}(\tau')|)}{(\tau - \tau')^2} \quad (25)$$

where:

$$\mathcal{K}(|\vec{R}|) \approx \frac{(Ua^3)^2}{2\pi^2(\hbar^2/m)^2} \frac{1 - \cos(2k_F|\vec{R}|)}{|\vec{R}|^2} \quad (26)$$

and we are assuming that  $k_F a \gg 1$ . We perform the  $\theta$  integration in eq.{9} using the saddle point approximation, and we obtain:

$$\alpha_n \sim \frac{(Ua^3)^2 k_F^2}{(\hbar^2/m)^2 (k_F R)^2} e^{-(3n^2)/(k_F R)^2} \quad (27)$$

and, finally, we can approximate:

$$\alpha_n \sim \frac{(Ua^3)^2 k_F}{(\hbar^2/m)^2 R} \sim \frac{\delta}{k_F R}, \quad n \ll k_F R \quad (28)$$

where we are defining the phaseshift  $\delta$  induced by the particle on the states near the Fermi level of the electron gas as:

$$\delta \sim \frac{(Ua^3)^2 k_F^6}{E_F^2} \quad (29)$$

Using eq.{28}, we obtain:

$$\begin{aligned} \sum_n n^2 \alpha_n &\sim \delta (k_F R)^3 \\ \sum_n n^4 \alpha_n &\sim \delta (k_F R)^5 \end{aligned} \quad (30)$$

and the parameter in eq.{12} is  $\kappa \sim (\hbar^2 k_F^2)/(ME_F) \ll 1$ .

Using equation {13}, we find:

$$E_C^{ren} \sim \frac{\hbar^2}{MR^2} e^{-2\pi^2 \delta} \quad (31)$$

Hence, the amplitude of the Aharonov-Bohm, as in the quantum rotor case, is reduced by a finite factor as  $R \rightarrow \infty$ . Similar results can be obtained using the response function of the two dimensional electron gas. The

qualitative features of the RPA response function used here are generic to the response of a clean electron gas, whose low energy excitations can be described in terms of Landau's theory. Finally, the results presented in this subsection remain valid if the local coupling between the particle and the electron gas is replaced by a screened electrostatic potential. If we assume that the charge of the particle is  $e^*$  and the charge of the electrons is  $e$ , the expression  $Ua^3$  in eq.{29} has to be replaced by  $e^*ek_{FT}$ , where  $k_{FT} = \sqrt{(4e^2mk_F)/(\pi\hbar^2)}$  is the Fermi-Thomas wavevector.

### E. Coupling to a dirty electron gas.

In this case, the susceptibility is given by:

$$\chi(\vec{k}, \omega) \approx \nu \frac{D|\vec{k}|^2}{i\omega + D|\vec{k}|^2} \quad (32)$$

We have defined, as in the previous subsection, the phase-shift as  $\delta \sim (U/E_F)^2(k_F a)^6$  and  $\nu \sim k_F^3/E_F$ . The terms in eq.{34} decay exponentially for values of  $\tau$  larger than  $E_T^{-1} \sim [(\hbar D)/R^2]^{-1}$ , where  $E_T$  can be defined as the Thouless energy for the electrons moving around paths comparable to the ring. Hence, in the more physical regime,  $E_T \gg E_C = \hbar^2/(MR^2)$ , or, alternatively,  $(k_F l)(M/m) \gg 1$ , the effect of the environment is exponentially suppressed, and there will be no significant renormalization of the Aharonov-Bohm oscillations.

ii) Coupling by the Coulomb potential.

In this case, one has to replace the product  $(Ua^3)\chi(\vec{k}, \omega)$  used previously by:

$$\text{Im} \left\{ \frac{4\pi e e^*}{|\vec{k}|^2 + 4\pi e e^* \chi(\vec{k}, \omega)} \right\} \approx \frac{|\omega|}{|\vec{k}|^2 D \nu} \quad (35)$$

where  $D$  is the diffusion coefficient,  $D \sim v_F l \sim (\hbar k_F l)/m$ ,  $l$  is the mean free path, and  $\nu$  is the density of states. This expression is valid for  $|\vec{k}| \ll l^{-1}$ , and  $\omega \ll \Lambda_0 \sim (\hbar D)/l^2$ . In the following, we consider separately the case when the coupling between the particle and the electrons is by means of a local potential, as generically described in eq.{6}, or by a screened Coulomb potential. Unlike in the case of a clean electron gas, discussed in the previous subsection, the two situations are not equivalent.

i) Coupling by a short range potential.

The time dependence of this kernel is not a simple power law. The function  $K(X, \tau)$  in eq.{6} becomes:

$$K(X, \tau) \sim \nu (Ua^3)^2 \frac{-D\tau + X^2}{\hbar \sqrt{D\tau} D^2 \tau^3} e^{-X^2/(D\tau)} \quad (33)$$

We can now take  $X = 2R \sin(\theta/2)$ , and decompose {33} in circular harmonics. Using the saddle point approximation to perform the integral over  $\theta$ , we obtain:

$$K_n(\tau) \sim \frac{\nu (Ua^3)^2}{\hbar D R} \left( \frac{1}{\tau^2} + \frac{n^2 D}{R^2 \tau} \right) e^{-(D\tau n^2)/R^2} \sim \frac{\delta}{k_F^2 R l} \left( \frac{1}{\tau^2} + \frac{n^2 D}{R^2 \tau} \right) e^{-(D\tau n^2)/R^2} \quad (34)$$

where  $e^*$  is the charge of the particle, and  $e$  that of the electrons. The kernel which describes the retarded interactions decays as  $\tau^{-2}$  at long times. The spatial dependence of the kernel  $\mathcal{K}$ , as defined in eq.{2}, becomes:

$$\mathcal{K}(X) \approx \int_{|\vec{k}| \ll l^{-1}} d^3 \vec{k} \frac{\sin(|\vec{k}|X)}{\hbar D \nu |\vec{k}|^3 X} \quad (36)$$

Setting  $X = 2R|\sin(\theta)/2|$ , and performing the Fourier transform defined in eq.{9}, we obtain:

$$\alpha_n \sim \int_{|\vec{k}| \ll l^{-1}} d^3 \vec{k} \frac{1}{|\vec{k}|^3 R} e^{-n^2/(\hbar |\vec{k}| R)^2} \sim \int_{k \sim n/R}^{k \sim 1/l} \frac{dk}{k} \sim \begin{cases} \frac{1}{\hbar \nu D R} \log\left(\frac{R}{nl}\right) & n \ll R/l \\ 0 & n \gg R/l \end{cases} \quad (37)$$

The parameter  $\kappa$  defined in eq.{12} is  $\kappa \sim (MR)/(\hbar \nu D l^2) \gg 1$ . The renormalization of  $E_C$  is, in

this case:

$$E_C^{ren} \sim \frac{\hbar^2}{MR^2} \left( \frac{l}{R} \right)^{c/(k_F l)^2} \quad (38)$$

where  $c$  is a constant of order unity.

#### IV. CONCLUSIONS.

We have analyzed the Aharonov-Bohm oscillations in five specific models for a particle coupled to different models of dissipative baths. In most cases, these oscillations are suppressed by a factor which can be written as  $e^{-c(\gamma l^2)/\hbar}$ , where  $\gamma$  is the macroscopic friction coefficient,  $l$  is a length which characterizes the spatial range of the interactions induced by the environment, and  $c$  is a constant. This factor is independent of the radius of the orbit,  $R$ . This is the case, for instance, when the particle is coupled to a clean electron gas, where  $l \sim k_F^{-1}$ .

However, when the environment is the Caldeira-Leggett bath of oscillators, the renormalization of the amplitude of the oscillations has a gaussian dependence on the radius of the circle in which the particle moves, and the suppression factor mentioned in the previous paragraph becomes  $e^{-c(\gamma R^2)/\hbar}$ . Hence, quantum interference effects become negligible beyond a certain length,  $R_\phi \sim \sqrt{\hbar/\gamma}$ , where  $\gamma$  is the friction coefficient. A less divergent suppression is also found for a charged particle coupled to a dirty electron gas, where the dependence of the Aharonov-Bohm amplitudes is a power law, although different from the value obtained in the absence of the environment.

This diverse behavior in different models arises from the spatial range of the retarded interaction induced by the environment. This difference is lost in the classical limit, which is attained at sufficiently high temperatures. When the thermal length,  $L_T \sim \sqrt{\hbar^2/(MT)}$ , is much shorter than the range of the retarded interaction, the effects of the environment can be expressed in terms of an effective friction coefficient, provided that the interaction

in time decays as  $\tau^{-2}$  at zero temperature, as in most cases considered here.

The suppression of interference effects, when it exists, is due to the formation of a screening cloud around the particle, with contributions from the high energy modes of the environment. The effect can be cast in terms of the existence of a Franck-Condon overlap factor which suppresses quantum interference effects<sup>19</sup>. This factor can depend on the length of the path of the particle around the magnetic flux, leading to the suppression of the Aharonov-Bohm oscillations. This interpretation is consistent with the fact that the same renormalization enters in the effective mass of the particle. The conductance can be defined as a function of the sensitivity of the ground state energy to a magnetic flux<sup>20</sup>. Hence, the Franck-Condon factor discussed here also reduces the conductance.

Finally, it is worth noting that the divergence of the renormalization of the effective mass which appears with the suppression of the Aharonov-Bohm oscillations implies a qualitative change in the propagator of the particle. The “quasiparticle peak” at zero momentum, which characterizes the propagator of a free particle in the ground state, is replaced by an incoherent background.

#### V. ACKNOWLEDGEMENTS.

I am thankful to Y. Imry, R. Jalabert, G. Schön and A. Zaikin for helpful conversations, and to A. Kamenev for pointing out to a mistake in an earlier version of the manuscript. This work was done while at the Institute for Theoretical Physics, Santa Barbara. This research was supported in part by the National Science Foundation under Grant No. PHY99-07949, and MEC (Spain) under Grant No. PB96/0875.

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